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| Lakes College West Cumbria |
| Electromagnetic Principles |
| Higher National Certificate in Engineering |

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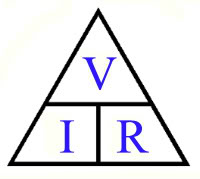
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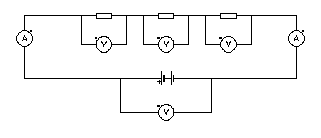
# Ohm’s Law

Ohms law states that current flowing in a circuit is directly proportional to the applied voltage.



# Resistors in Series

The 3 resistors R1, R2 and R3 are connected in series with a battery. When the circuit is closed a current will flow, this will be the same in all parts of the circuit.



**R1**

**R2**

**R3**

**I**

**V**

**V1**

**V2**

**V3**

V = V1 + V2 + V3

From Ohms Law

V1 = IR1, V2 = IR2, V3 = IR3 and V = IR

Where R is the total circuit resistance

Since V = V1 + V2 + V3

then IR = IR1 + IR2 + IR3

Dividing throughout by I gives:

R = R1 + R2 + R3 Total Resistance in series

Problem

For the circuit shown below, determine the p.d. across resistor R3. If the total resistance of the circuit is 100Ω, determine the current flowing through resistor R1. Find also the value of resistor R2.

10V

4V

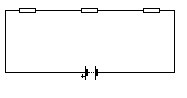
V3

25V

R1

R2

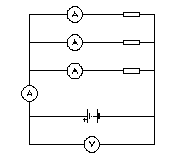
R3



Solution: -

# Resistors in Parallel

The circuit below shows three resistors R1, R2 and R3 connected across each other, i.e. in parallel, across a battery source of V volts.



R1

R2

R3

I1

I2

I3

**V**

In a parallel circuit:

The sum of currents I1, I2 and I3 is equal to the total current, I,

i.e. I = I1 + I2 + I3

The source p.d., V volts, is the same across each of the resistors.

From Ohm’s law:

I1 = V , I2 = V , I3 = V and I = V

R1 R2 R3 R

Where ‘R’ is the total circuit resistance. Since

I = I1 + I2 + I3

Then



Dividing throughout by V gives

 Resistances in Parallel

For the **special case of two resistors in parallel**

1 = 1 + 1\_ = R2 + R1

R R1 R2 R1\* R2

Hence,

R = R1 \* R2 (i.e. product )

R1 + R2 sum

Problem

For the circuit shown, find (a) the value of the supply voltage, V, and (b) the value of current, I. Check your answer using MultiSim.



I2 = 3A

R3

60Ω

R2

R1

20Ω

10Ω

V

Solution:

# Electrical Power

Power is defined as the rate of doing work, or transferring energy.

Power (*P)* in an electrical circuit is given by the product of potential difference (*V)* and current (*I).* The unit of power is the Watt (*W)*.

**P = V \* I**

From Ohms law V = I \* R. Hence

P = (I\*R) \* I

**P = I2\*R**

Also from Ohms law I = V/R. Hence

P = V \* (V/R)

**P = V2**

**R**

### Problems

1. Calculate the power dissipated when a current of 4mA flows through a resistance of 5kΩ
2. A resistor has a power rating of 5W. If it is connected to a 1.5V source, what is the maximum current flow permitted?
3. A current of 5A flows in the winding of an electric motor, the resistance of the winding being 100Ω. Determine
   1. The p.d. across the winding
   2. The power dissipated by the coil

# cartoon_krchoff3.gifKirchhoff’s Laws

## Introduction

The laws that determine the currents and voltage drops in d.c. networks are:

1. Ohms Law
2. The laws for resistors in series and parallel
3. Kirchhoff’s laws

## Kirchhoff’s First Law

At any junction in an electric circuit the total current flowing toward that junction is equal to the total current flowing away from the junction i.e. *ΣI=0*

Therefore

I1 + I2 = I3 + I4 + I5 OR I1 + I2 - I3 - I4 - I5  =0

I5

I4

I3

I2

I1

## Second Law

In any closed loop in a network, the algebraic sum of the voltage drops taken around the loop is equal to the resultant e.m.f. acting in that loop.

Thus in the circuit

E1 – E2 = IR1 + IR2 + IR3

I

R2

R3

R1

E2

E1

### Worked Example

Use Kirchhoff’s laws to determine the currents flowing in each branch of the network.

r1 = 2Ω

r2 = 1Ω

E1 = 4V

R = 4Ω

E2 = 2V

**1st Law**

Use Kirchhoff’s law and label current directions on the original circuit diagram.

It is usual to assume that the current flows from the positive terminals of the batteries.

I1+I2

I2

r1 = 2Ω

r2 = 1Ω

E1 = 4V

R = 4Ω

I1

E2 = 2V

Loop 2

Loop 1

**2nd Law**

Divide the circuit into 2 loops and apply Kirchhoff’s voltage law to each.

In Loop 1

E1 = I1r1 + (I1+I2)R

4 = 2I1 + 4(I1+I2)

4 = 6I1 + 4I2 (1)

In Loop 2

E2 = I2r2 + (I1+I2)R

2 = I2 + 4(I1+I2)

2 = 4I1 + 5I2 (2)

Problem 4

Use Kirchhoff’s laws to find the equations for voltage drop in each loop.

50Ω

100Ω

80V

80Ω

100V

Problem 5

For the circuit shown calculate I1, I2  and I3.

1

I3

I2

I1

Problem 6

Solve for the voltage across the load resistor RL.



# Thevenin’s Theorem

Using Kirchhoff’s laws to solve the circuit in figure 1 can be done, but will be time consuming. However the use of Thevenin’s theorem enables us to replace the active network with a single equivalent circuit. It is then possible to replace the load and quickly calculate its effect on the circuit.

Figure 1 – Active network and load resistances

RA

RC

RE

RB

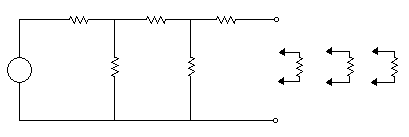
RD

E

R1

R2

R3



Thevenin’s theorem tells us that in any linear active network having output terminals A and B behaves as far as measurements at the output terminals are concerned as though it consisted of a single constant-voltage of e.m.f. E volts in series with a single passive impedance RG as shown if figure 2.

Figure 2 – Thevenin’s equivalent

A

RG



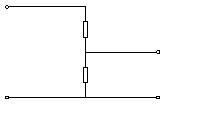
RL

E

B

B

Potential Divider Circuit



R1

VIN

VOUT

R2

This circuit is often referred to as a potential divider circuit.

Such a circuit can consist of a number of similar elements in series connected across a voltage source, voltage being taken from connections between the elements. Frequently the divider consists of two resistors:

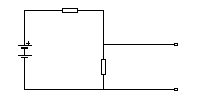
VOUT = R2 \* VIN

R1 + R2

Example – Potential Divider

Determine the value of voltage in the circuit shown below.

4Ω

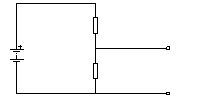


50V

6Ω

V

This circuit may be redrawn as:



4Ω

50V

6Ω

V

Now, voltage V = 6Ω \* 50V = 30V

6Ω + 4Ω

## 

## Worked Example

Obtain the Thevenin’s equivalent of the network shown below at the terminals A-B. Find the terminal p.d. and the current flowing when a 27Ω resistor is connected across A-B.

4Ω

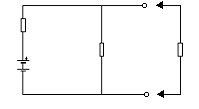
12Ω

A

B

RL = 27Ω

10V



Step 1

EOC = p.d. across the 12Ω resistor (and the terminals) with the load removed.

Therefore,

EOC = 10 \* 12 = 7.5V

4 + 12

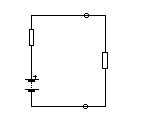
Step 2

Impedance seen looking into terminals A-B with the battery shorted out

RG = 4 \* 12 = 3Ω

4 + 12

A



3Ω

Equivalent Circuit

When RL= 27Ω, the output current is

27Ω

7.5VV

***IL=E/(R+r)***

IL = 7.5V = 0.25A

30Ω

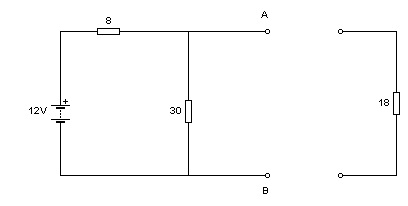
B

and the p.d. across the load = 27Ω \* 0.25A = 6.75V

## Problems

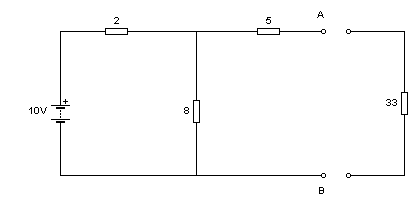
Q. 7

Find the Thevenin equivalent circuit for the network shown below. Hence find the p.d. across an 18Ω load resistor connected across A-B



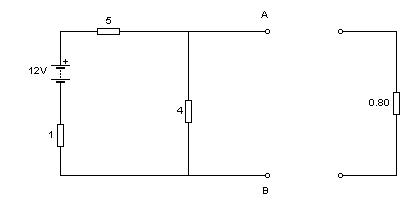
Q. 8

Find the Thevenin equivalent circuit for the network shown below. Hence find the current that would flow in a 33Ω load resistor connected across A-B



Q. 9

Find the Thevenin equivalent circuit for the network shown below. Hence find the p.d. across a 0.8Ω load resistor connected across A-B.



# Norton’s Theorem

Norton’s theorem states that in a linear network of generators and impedances the output terminals A and B as shown in figure 1 (a) can be replaced by a single constant-current generator in parallel with a single impedance RG as shown in ‘figure 1 (b).

Figure 1 – Nortons equivalent circuit

Network of generators

and impedances

A

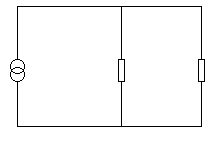
B



RL

(a)

A



B

(b)

RL

RG

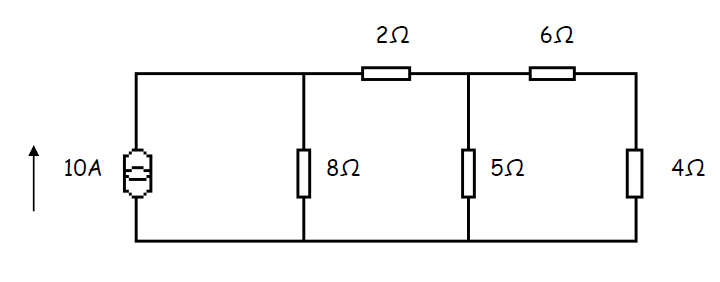
I

The equivalent current source ISC is that current which flows in a short-circuit placed across the terminals of the network. The internal impedance of this source is infinite, but has in parallel with it an impedance of the network seen looking back into the terminals A-B.

This theorem is simply a dual of Thevenin’s theorem in that it enables a network to be replaced by a single generator and impedance. In this instance, however, the generator is of the constant-current type and the impedance is in parallel with it, whereas in Thevenin’s theorem the equivalent circuit is of a constant-voltage generator in series with the impedance.

## Worked Example

Determine the current flowing through the 5Ω resistor.



Step 1: Calculate the short circuit current

Consider there to be terminals on either side of the 5Ω resistor. Then short-circuit the terminals as shown.

Step 2: Calculate the internal resistance

Draw Norton Equivalent circuit

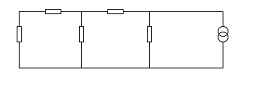
## Norton Problems

Q.10

Determine the current flowing in the 6Ω resistance of the network shown below using Norton’s theorem.

8Ω

2Ω



10Ω

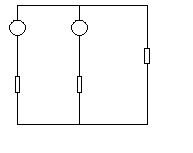
6Ω

4Ω

6mA

Q. 11

Use Norton’s theorem to determine the current, I, flowing in the 4Ω resistance shown below:



4V

2V

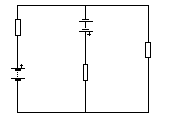
4Ω

2Ω

1Ω

Q. 12

Determine the current in the 5Ω resistance of the network using Nortons theorem.



0.5Ω

12V

5Ω

2Ω

4V

# Maximum Power Transfer

Problem 1

The circuit diagram shows a source of 6V with an internal resistance of 2.5Ω.

The load resistance is varied from 0 to 5Ω in 0.5Ω steps.

Calculate the power dissipated by the load in each case.

Plot a graph of RL (horizontally) against power (vertically) and determine the maximum power dissipated.

I

E = 6V

R = 2.5Ω

Load

RL

|  |  |  |
| --- | --- | --- |
| **RL** | **I=E/(r+RL)** | **P=I2RL** |
| 0 |  |  |
| 0.5 |  |  |
| 1.0 |  |  |
| 1.5 |  |  |
| 2.0 |  |  |
| 2.5 |  |  |
| 3.0 |  |  |
| 3.5 |  |  |
| 4.0 |  |  |
| 4.5 |  |  |
| 5.0 |  |  |

Problem 13

A d.c. source has an open-circuit voltage of 20V and an internal resistance of 2Ω. Determine the value of the load resistance that gives maximum power dissipation. Find the value of this power.

Problem 14

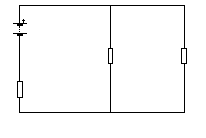
A d.c. source has an open circuit voltage of 30V and an internal resistance of 1.5Ω. State the value of load resistance that gives maximum power dissipation and determine the value of this power.

Problem 15

A generator has an open-circuit voltage of 12V and an internal resistance of 40Ω. Calculate the load resistance for maximum power transfer. Find the value of this power.

Problem 16

Determine the value of the load resistance RL, shown below that gives maximum power dissipation and find the value of the power.



24V

RL

8Ω

2Ω

# Charging and Discharging a Capacitor

# Introduction

When a d.c. voltage is applied to a capacitor C and a resistor R connected in series, there is short period of time immediately after the voltage is connected during which the current flowing in the circuit and voltages across C and R are changing.

These changing values are called transients.

## Charging a capacitor

**R**

### C

VR

Vc

### V

We know from Kirchoff’s voltage law that V = Vc + VR.

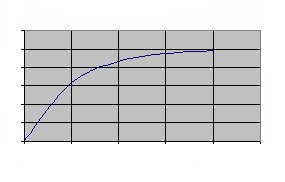
Assuming that there is no initial charge on the capacitor when the battery is connected Vc = 0. Therefore VR = V and the initial current I = V/R.

After t1 seconds the capacitor is partly charged because a current has been flowing. If the current flowing is i1 then the voltage drop across R = i1R.

After t2 seconds the capacitor continues to charge. The voltage across the capacitor Vc has increased and VR has decreased to i2R.

Ultimately the capacitor is fully charged and the current no longer flows. As I = 0, VR = 0. Therefore V= Vc.

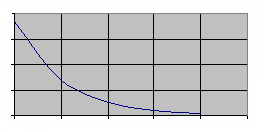
### Exponential Growth and Decay Curves

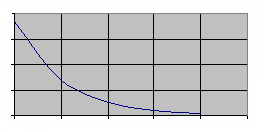
The curves representing the variation of Vc with time is called the exponential growth curve.

Capacitor Voltage

(V)



The curves showing the variation of VR and I with time are called exponential decay curves.



Resistor

Voltage

(V)

Current

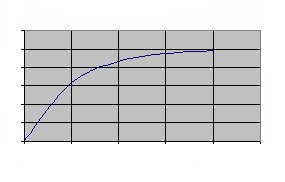
(A)

Time (s)

Time (s)

## Time constant for a C-R circuit

If the constant voltage supply is replaced by a variable voltage supply, and at time t1 seconds the voltage is varied so that the current remains constant, then the capacitor voltage will reach it’s final value at time t2 seconds.



The time corresponding to (t2-t1) seconds is called the time constant of the circuit.

Vc

**Time constant** τ **= CR seconds**

t(s)

For a growth curve, the value of a transient:

* At time equal to one time constant is 0.632 of its steady state value
* At time equal to 2½ time constants is 0.918 of its steady state value
* At time equal to 5 time constants is its steady state value

For a decay curve, the value of a transient:

* At time equal to one time constant is 0.368 of its initial value
* At time equal to 2½ time constants is 0.082 of its initial value
* At time equal to 5 time constants zero

The equations of the curves are:

Growth of capacitor voltage:

**Vc = V(1 – e –t/CR) = V(1 – e –t/**τ **)**

Decay of resistor voltage**:**

**VR = Ve –t/CR = Ve –t/**τ

Decay of current:

**i = Ie –t/CR = Ie –t/**τ

## Discharging a Capacitor

When a capacitor is charged and the voltage supply switched out of the circuit, the electrons stored in the capacitor keep the current flowing for a short time.



VR

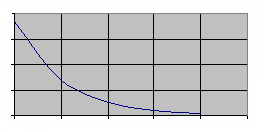
Vc

Initially the current flow is such that the capacitor voltage Vc is balanced by an equal and opposite voltage VR = iR. Since initially Vc = VR = V, then i=V/R.

During the transient decay Vc = VR

Finally the transients decay exponentially to zero.

### Exponential decay curves



Time (s)

Current

(A)

Capacitor

And

Resistor

Voltage

(V)

Time (s)

The equations of the curves are:

Decay voltage**:**

**Vc = VR = Ve –t/CR = Ve –t/**τ

Decay of current:

**i = Ie –t/CR = Ie –t/**τ

Note: When a capacitor has been disconnected from the supply it may still be charged and it may retain this charge for some time. Thus precautions must be taken to ensure that the capacitor is automatically discharged after the supply is switched off. This is done by connecting a high value resistor across the capacitor terminals.

### Calculations

Example

A 20μF capacitor is connected in series with a 50kΩ resistor and the circuit is connected to a 20V d.c. supply. Determine

1. The initial value of the current
2. The time constant of the circuit
3. The value of the current 1second after connection
4. The value of the capacitor voltage 2 seconds after connection
5. The initial value of the current flowing is:

I = V = 20 = 0.4mA

R 50 \* 103

1. The time constant τ = C \* R = (20 \* 106) \* ( 50 \* 103) = 1s
2. Current i = Ie-t/τ = 0.4e-1/1 = 0.4 \* 0.368 = 0.147mA
3. Capacitor Voltage

Vc = V (1 - e-t/τ)

= 20 (1 - e-2/1)

= 20 (1 – 0.135)

= 20 \* 0.865

= 17.3V

Problem 17

A circuit consists of a resistor connected in series with a 0.5μF capacitor and has time constant of 12mS. Determine:

1. The value of the resistor
2. The capacitor voltage 7mS after connecting the circuit to a 10V supply.

Problem 18

A 0.1μ capacitor is charged to 200V before being connected across a 4kΩ resistor. Determine

1. The initial discharge current
2. The time constant of the circuit
3. The minimum time for the voltage across the capacitor to fall to less than 2V

# Charging and Disconnecting an Inductor

## Introduction

When a d.c. voltage is connected to a circuit having an inductance L connected in series with a resistance R, there is a short period of time immediately after the voltage is connected, during which the current flowing in the circuit and the voltages across L and R are changing.

These changing values are called transients.

## Current growth in an L-R circuit



VR

VL

### V

We know from Kirchoff’s voltage law that V = VL + VR.

The battery voltage V is constant. The voltage across the inductance is the induced voltage:

**VL = L \* change of current = L d*i***

**change of time d*t***

The voltage drop across R is given by iR.

Hence:

**V = L d*i* + iR**

**d*t***

When the battery is connected the rate of change of current is such that it induces an e.m.f. in the inductance which is equal and opposite to V.

V = VL and VR = 0.

After t1 seconds current i1 is flowing resulting in a voltage drop of i1R across the resistor. This gives:

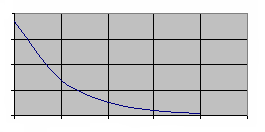
V = L d*i*1 + i1R

d*t*1

After t2 seconds the current flowing is i2, and the voltage drop across the resistor increases to i2R. Since VR increases VL decreases.

Ultimately the current flow is entirely limited by R. The rate of change of current is zero and hence VL is zero. Therefore V=iR.

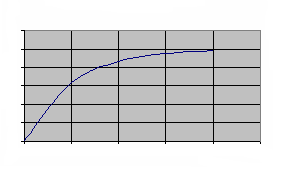
### Exponential Growth and Decay Curves

The curves representing the variation of VL with time is an exponential decay curve.

Induced Voltage

(V)

Time (s)

The curves showing the variation of VR and I with time are exponential growth curves.

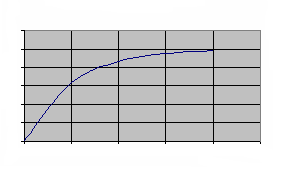
Resistor

Voltage

(V)

Current

(A)



Time (s)

Time (s)

## Time constant for a L-R circuit

The time constant of a series connected L-R circuit is given by:

**Time constant τ = L seconds**

**R**

The equations of the curves are:

Decay of induced voltage:

**VL = Ve –Rt/L = Ve –t/**τ

Growth of resistor voltage**:**

**VR = V(1 – e – Rt/L) = V(1 – e –t/**τ **)**

Growth of current flow:

**i = I(1 – e – Rt/L) = I(1 – e –t/**τ **)**

### Current decay in an L-R circuit

When a series connected L-R circuit is connected to a d.c. supply, with the switch in position A, a current I = V/R flows after a short time creating a magnetic field associated with the inductor.



VR

VL

A

B

B

When the switch is moved to position B the current value decreases; this causes a decrease in the strength of the magnetic field. This generates a voltage VL = d*i*

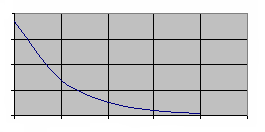
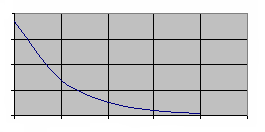
d*t*

The value of R limits the value of the current in the circuit.

As current decays exponentially to zero, VR also decays exponentially to zero.

Since VL = VR, VL also decays exponentially to zero.

### Exponential decay curves



Current

(A)

Inductor

And

Resistor

Voltage

(V)

Time (s)

Time (s)

The equations of the curves are:

Decay voltage**:**

**VL = VR = Ve –Rt/L = Ve –t/**τ

Decay of current:

**i = Ie –Rt/L = Ie –t/**τ

### Problems

Problem 19

A coil of inductance 0.04 H and resistance 10Ω is connected to a 120V d.c. supply. Determine:

1. The final value of the current
2. The time constant of the circuit
3. The value of the current after a time equal to the time constant from the instant the supply voltage is connected.
4. The expected time for the current to rise within 1% of its final value.

Problem 20

An inductor has a negligible resistance and an inductance of 200mH and is connected in series with a 1kΩ resistor to a 24V d.c. supply. Determine the time constant of the circuit and the steady state value of the current flowing in the circuit.

Find:

1. The current flowing in the circuit at a time equal to one time constant
2. The voltage drop across the inductor at a time equal to two time constants
3. The voltage drop a across the resistor at a time equal to three time constants

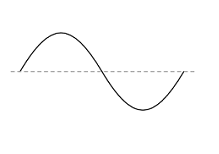
Problem 21

The field winding of a 200V d.c. machine has a resistance of 20Ω and an inductance of 500mH. Calculate:

1. The time constant of the field winding
2. The value of the current flow one time constant after being connected to the supply
3. The current flowing 50ms after the supply has been switched on.

# The equation of a sinusoidal waveform

Vp

The time taken for an alternating quantity to complete one 1 cycle is called the periodic time, T, of the waveform. The number of cycles completed in one second is called the frequency, f, of the supply and is measured in Hertz (Hz)

2π

3π

2

π

2

π

time = 1 ∴ freq = 1

freq time

Problems

1. Determine the periodic time for the following frequencies:

1. 2.5Hz
2. 100Hz
3. 40kHz

2. Calculate the frequency for the following periodic times:

1. 5ms
2. 50μs
3. 0.2s

v = Vpsinωt

Vp = peak value (volts)

ω = 2πf = angular velocity (rad/s)

t = time (s)

## Example Calculation

An alternating voltage waveform is represented by the expression



Determine

1. The peak value;
2. The frequency;
3. The period of the waveform;
4. The value at t = 3.5ms

### Problems

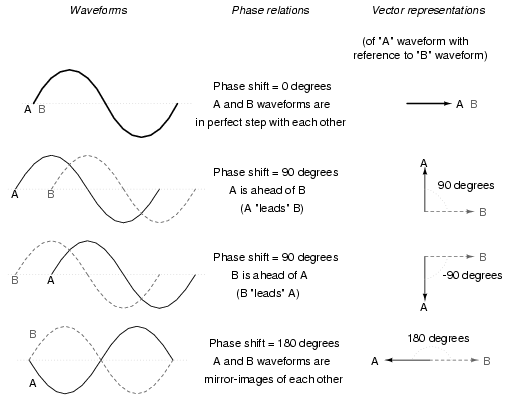
1. An alternating voltage is given by v = 75 sin (200πt). Find

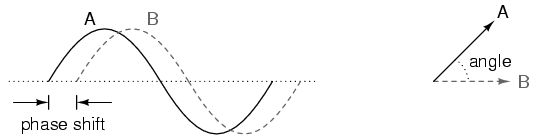
1. the amplitude
2. the peak-to-peak value
3. the rms value
4. the frequency
5. the periodic time

2. A sinusoidal current has a peak value of 30A and a frequency of 60Hz. At time t=0 the current is zero. Express the current in the form i = Ipsinωt

## Phase and Phase Angle

A sine wave may not always start at 0°





The general sinusoidal voltage waveform is now

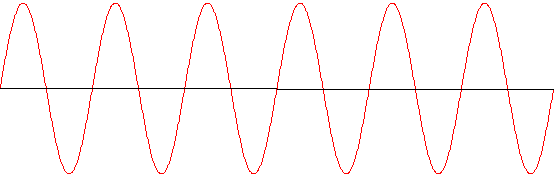
v = Vp sin(ωt ±φ)

where φ = angle of lag or lead compared with v= Vpsin(ωt)

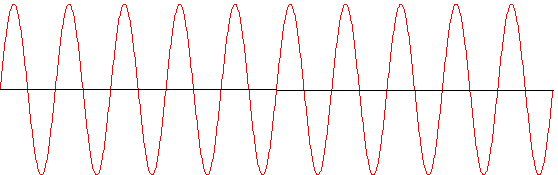
## Complex Waves

A complex wave varies from instant to instant, but can be resolved into a number of sine-wave components, each of a different frequency.

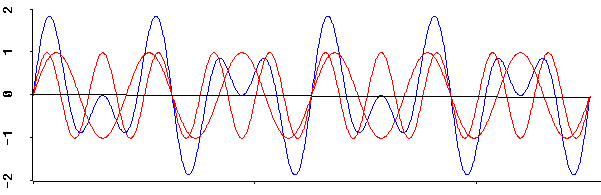
1st sine wave



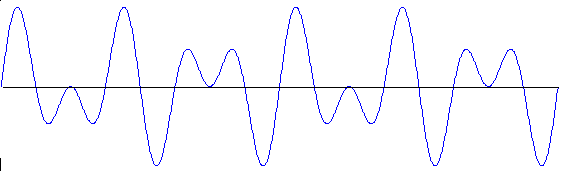
2nd sine wave



They can be added together

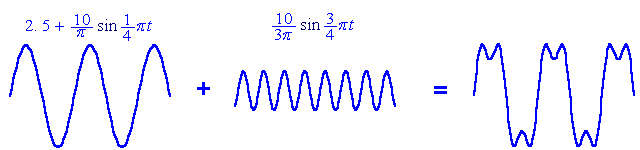


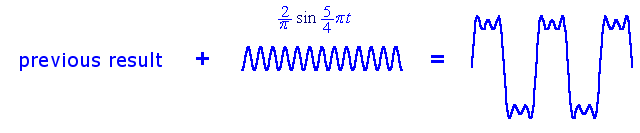
To form a complex wave

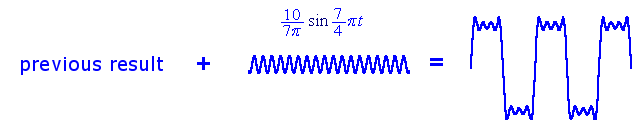


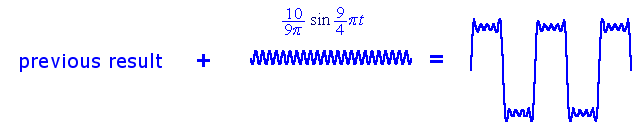
Any complex wave can be treated as a combination of simple sine waves.

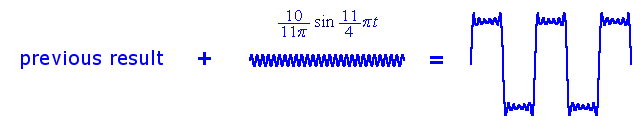
If we are trying to map a square wave we can split it into an infinite series of sine waves as seen below.







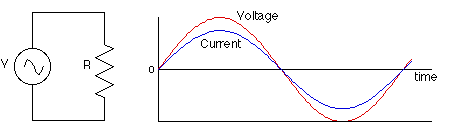




# Series AC Circuits

## Circuits Containing Resistance Only

When an alternating voltage is applied to a resistive circuit (a circuit containing resistance only), the current (i) is in phase with the applied voltage (v).

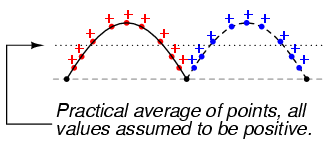


If the voltage at any instant is v volts, the value of current at that instant is given by:

When the voltage is zero, the current is also zero; and since the current is proportional to the voltage, the waveform of the current and voltage are exactly the same. They also reach the maximum values at the same time. If Vm and Im are the maximum values of voltage and current respectively, therefore:-

## RMS Value

However r.m.s. values are used when dealing with alternating current circuits. This is equivalent to that value of direct current that will dissipate the same amount of power.



The r.m.s. value of a sine wave is 0.707 times the maximum value, so that:

r.m.s. value of voltage V = 0.707 Vm

r.m.s. value of current I = 0.707 Im

Substituting for Im and Vm therefore:-

Hence ‘ohms law’ can be applied without modification to an AC circuit containing resistance only.

**Phasor Diagram** representing the voltage and current in a resistive circuit.

V

I

The two phasors are actually coincident, but are actually drawn slightly apart so that the identity of each may be clearly recognised. It is usual to draw the phasors in the position corresponding to θ = 0 (as there is no phase shift between voltage and current), therefore the phasors are drawn along the x-axis.

## Problems

### Circuits containing resistance only

Q.1.

A resistance of 10Ω is connected across an AC supply of 240V, calculate the circuit current.

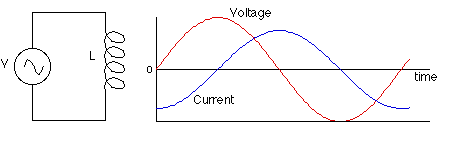
Q.2.

An unknown resistance is connected across an AC supply of 100V,and a current of 2Amperes flows. Calculate the resistance value.

Q.3.

An unknown resistance is connected across an AC supply of 200V and a current of 5A flows. Calculate the resistance value.

## Circuits containing Inductance only



This is a purely theoretical consideration because all inductances (coils) must have some resistance since they are made of wire.

When an alternating current is flowing in a circuit containing inductance the current is changing constantly, so the inductance will produce a constantly changing magnetic field. When a wire is situated in a changing magnetic field it will produce a back e.m.f. (voltage) which is in opposition to the applied voltage, therefore limiting the amount of current flowing within the circuit. This limitation in current is caused by what is known as Inductive reactance.

Inductive reactance is the opposition to current flow (similar to resistance in a resistive circuit) in an inductive circuit and is given the symbol XL, the unit is the Ω (ohms).

ω represents angular velocity

L represents inductance

f represents frequency

2π represents one cycle

When calculating current in an inductive circuit ‘ohms law’ still applies except that resistance is replaced by Inductive reactance XL.

When an alternating voltage is applied to an inductive circuit ( a circuit containing inductance only), the current lags the applied voltage by 90°.

The current lags the voltage by 90° because it takes time for the magnetic field to establish within the inductor. The phase difference corresponds to one quarter of a cycle.

**Phasor Diagram** representing voltage and current in an inductive circuit.

V

I

## Problems

### Circuits containing Inductance only

Q.4.

Calculate the inductive reactance when a 10 Henry inductance is connected to an AC supply with a frequency of 50Hertz.

Q.5.

A coil of inductance 0.2H and negligible resistance is connected across an AC supply of 230V, 50Hz. Calculate:

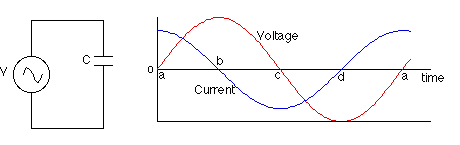
1. The inductive reactance
2. The current within the coil

Q.6.

A coil of inductance 0.1H and negligible resistance is connected across an AC supply of 110V, 60Hz. Find:

1. The inductive reactance
2. The current flowing within the circuit.

## Circuits containing Capacitance only



When a direct current is applied to a capacitor, current flows whilst the capacitor is charging or discharging. When an alternating current is applied to a capacitor the capacitor is constantly charging and discharging as the alternating current changes value and direction. The maximum current flows in a capacitive circuit when the capacitor contains no charge; the minimum current flows when the capacitor has maximum charge. Therefore current will be out of phase with the applied voltage. As a capacitor is constantly charging and discharging, this will limit the current flow within the circuit.

Capacitive Reactance is the opposition to current flow in a circuit containing capacitance, and is given the symbol XC; the unit is the Ω (ohm).

C represents capacitance

ω represents angular velocity

2π represents one cycle

f represents frequency

When calculating current in a capacitive circuit ‘ohms law’ still applies; except that resistance R is replaced by Capacitive Reactance XC.

When an alternating current is applied to a capacitive circuit (a circuit containing capacitance only), the current leads the applied voltage by 90°. It can be seen from the diagram that this is the opposite to an inductive circuit.

The current leads the voltage by 90°, because with a capacitor the current is at its maximum when the voltage is at its minimum.

**Phasor Diagram** representing voltage and current in an inductive circuit.

I

V

## Problems

### Circuits containing Capacitance only

Q.7.

A capacitor is 0.02F is connected to an AC supply with a frequency of 50Hz. Calculate the capacitive reactance of the circuit.

Q.8.

A 50µF capacitor is connected across a 250V AC supply with a frequency of 50Hz. Calculate the current flowing within the circuit.

Q.9.

A 400µF capacitor is connected across a 415V AC supply with a frequency of 50Hz. Find the circuit current.

# AC Series circuits containing Resistance and Inductance

### VL VR V Circuit Diagram for R-L Series Circuit

The above diagram shows the potential differences (Voltage drops) within the circuit, VR volt drop due to resistance and VL volt drop due to inductance. The circuit contains inductance therefore there will be a phase angle between the current and the applied voltage.

**Method 1**: using a phasor diagram drawn to scale.

Step 1: draw a line on the horizontal reference axis from 0 to represent the current (Note: current is the common factor for a series circuit), then draw a line to a scale vertically upwards from 0, to represent VL. This is done because VL has a phase angle of 90°

VL

I

Step 2: Draw VR on the horizontal axis. VR is drawnon this axis because it is in phase with the current I.

VL

I

VR

Step 3: Complete the parallelogram using the values of VR and VL and draw from 0 the diagonal shown in the diagram. The diagonal represents the applied voltage, which is the resultant of VR and VL.

VL

VR

V

θ

I

In an AC circuit containing resistance R and inductance L in series, the applied voltage V is the phasor sum of VR and VL. The current I lags the applied voltage by an angle lying between 0° and 90°, depending upon the values of VR and VL, which in turn depends on the values of R and L. The circuit phase angle is the angle Φ between the current and the applied voltage. In any series circuit the current is the common factor and is common to all components within the circuit.

### Phase angle Φ

This is the angle between the current and the applied voltage and can be found by measurement using a protractor or by calculating using trigonometry.

For circuits containing resistance and inductance only, the phase angle is between 0° and 90°. The actual phase angle depends on the values of resistance and inductive reactance of the circuit.

### Impedance

The amount of current that flows in the circuit is affected by both resistance and inductive reactive. The total opposition to current flow is called impedance, symbol Z, unit Ω (ohm).

The impedance can be calculated using ohms law:-

Impedance may also be found using the impedance triangle:

Resistance R

Impedance Z

Φ

Inductive Reactance XL

The inductive reactance of the circuit can be calculated by dividing the voltage across the inductor by the circuit current.

The resistance of the circuit can be calculated by dividing the voltage across the resistance by the circuit current.

**Method 2**: Using Calculations

Voltage and Impedance can also be calculated by using Pythagoras’s theorem:

For impedance this is:

The impedance triangle may also be used to calculate the phase angle of the circuit i.e:

Similarly for voltage

VL

VR

V

θ

I

It is recommended that a rough phasor diagram is constructed to check that the answers are in the right ball park.

## Problems

## Circuits containing Resistance and Inductance

Q.10

A circuit of inductance 0.1H and resistance 30Ω, connected in series across a 100V, 50Hz AC supply. Draw the phasor diagram. Calculate the circuit impedance, the circuit current, the phase angle, and the potential difference across each component.

Q.11

A circuit of inductance 0.2H and resistance 40Ω, connected in series across a 100V, 50Hz AC supply. Draw the phasor diagram. Calculate the circuit impedance, the circuit current, the phase angle and the potential difference across each component.

# AC Series circuits containing Resistance and Capacitance



VC

VR

V

The above diagram shows the potential differences (voltage drops) within the circuit; VR volt drop due to resistance and VC volt drop due to capacitance. The circuit contains capacitance and therefore there will be a phase angle between the current and the applied voltage.

**Method 1**: using a phasor diagram drawn to scale.

I

Step 1: draw a line on the horizontal reference axis from 0 to represent the current (Note: current is the common factor for a series circuit), then draw a line to a scale vertically downwards from 0, to represent VC. This is done because VC has a phase angle of -90°

VC

VR

VC

I

Step 2: Draw VR on the horizontal axis. VR is drawnon this axis because it is in phase with the current I.

α

VR

VC

V

I

Step 3: Complete the parallelogram using the values of VR and VC and draw from 0 the diagonal shown in the diagram. The diagonal represents the applied voltage, which is the resultant of VR and VC.

In an AC circuit containing resistance R and capacitance C in series, the applied voltage V is the phasor sum of VR and VC. The current I lags the applied voltage by an angle lying between 0° and -90°, depending upon the values of VR and VC, which in turn depends on the values of R and C. The circuit phase angle is the angle Φ between the current and the applied voltage. In any series circuit the current is the common factor and is common to all components within the circuit.

### Phase angle Φ

This is the angle between the current and the applied voltage and can be found by measurement using a protractor or by calculating using trigonometry.

For circuits containing resistance and capacitance only, the phase angle is between 0° and -90°. The actual phase angle depends on the values of resistance and capacitive reactance of the circuit.

### Impedance

Both resistance and capacitive reactance affect the current flowing in a circuit. The total opposition to current flow is called impedance, symbol Z, unit Ω (ohm).

The impedance can be calculated using ohms law:-

Impedance may also be found using the impedance triangle:

Resistance R

Impedance Z

Φ

Capacitive Reactance XC

The capacitive reactance of the circuit can be calculated by dividing the voltage across the capacitor by the circuit current.

The resistance of the circuit can be calculated by dividing the voltage across the resistance by the circuit current.

**Method 2**: Using Calculations

Voltage and Impedance can also be calculated by using Pythagoras’s theorem:

For impedance this is:

The impedance triangle may also be used to calculate the phase angle of the circuit i.e:

Similarly for voltage

VR

I

V applied Voltage

VC

Φ

It is recommended that a rough phasor diagram is constructed to check that the answers are in the right ball park.

# Problems

## Circuits containing Resistance and Capacitance

Q.12

A circuit of capacitance 100µF and resistance 20Ω, connected in series across a 120V, 50Hz AC supply. Draw the phasor diagram. Calculate the circuit impedance, the circuit current, the phase angle, and the potential difference across each component.

Q.13

A circuit of capacitance 50µF and resistance 40Ω, connected in series across a 120V, 50Hz AC supply. Draw the phasor diagram. Calculate the circuit impedance, the circuit current, the phase angle and the potential difference across each component.

# AC Series Circuits containing Resistance, Inductance and Capacitance



V

VR

VL

VC

The above diagram shows the potential differences (voltage drops) within the circuit, VR volt drop due to resistance, VL volt drop due to inductance and VC volt drop dueto capacitance. The phase angle, i.e. the angle between the current and the applied voltage, depends upon the values XL and XC.

If XL is greater than XC the current will lag the applied voltage and the circuit will behave as if it were an R-L series circuit.

If XC is greater than XL the current will lead the applied voltage and the circuit will behave as if it were an R-C series circuit.

**Phasor diagram** for condition when XL is greater than XC.

The current is placed on the reference axis with VR in phase with it. The remaining two voltages VL and VC are drawn so that I lags VL by 90°, and VC by 90°

VL

VC

I

VR

VL

VC

I

VR

VL-VC

Find the resultant of VL and VC

VL

φ

VC

V

I

VR

VL-VC

Construct a parallelogram to find the applied voltage and phase angle

**Phasor diagram** for condition when XC is greater than XL.

VL

VC-VL

VR

I

VC

The phasor diagram is identical except that VC is greater than VL. Therefore that resultant is negative.

VL

α

V

VC-VR

VR

I

VC

In a ac circuit containing resistance, inductance and capacitance the applied voltage V is the phasor sum of VR, VL and VC. The current will lag the applied voltage V by an angle between 0° and 90° if VL is greater than VC. The current will lead the applied voltage V by an angle between 0° and -90° if VC is greater than VL. In a series circuit the current is the common factor and is common to all components within the circuit.

### Circuit Impedance for R-L-C Series Circuits

Resistance, Inductive reactance and Capacitive reactance affect the current flowing in a circuit. The formula used here is a slightly modified version of the equations for R-L and R-C series circuits, which have already been investigated.

The reactance is always the difference between XL and XC and is called the Resultant Reactance.

Using the impedance triangle

XL > XC

Resistance R

Impedance Z

Φ



 and 

Resistance R

Impedance Z

Φ

Resultant Reactance

XC – XL

 and 

# Problems

## Circuits containing Resistance, Inductance and Capacitance

Q.14

A circuit contains a resistance of 50Ω, and inductive reactance of 70Ω and a capacitive reactance of 30Ω, connected in series across an AC supply of 230V, 50Hz. Calculate circuit impedance, circuit current, phase angle, the potential difference across each component, and construct a phasor diagram.

Q.15

A circuit contains a resistance of 20Ω, an inductive reactance of 50Ω and a capacitive reactance of 30Ω, connected in series across an Ac supply of 230V, 50Hz. Calculate circuit impedance, circuit current, phase angle, the potential difference across each component, and construct a phasor diagram.

Q.16

A circuit contains a resistance of 10Ω, an inductive reactance of 50Ω and a capacitive reactance of 70Ω, connected in series across an AC supply of 230V, 50Hz. Calculate circuit impedance, circuit current, phase angle, the potential difference across each component, and construct a phasor diagram.

Q.17

A circuit contains a resistance of 30Ω, an inductive reactance of 50Ω, and a capacitive reactance of 70Ω, connected in series across an AC supply of 230V, 50Hz. Calculate circuit impedance, circuit current, phase angle, the potential difference across each component, and construct a phasor diagram.

Series Resonance



V

VR

VL

VC

Series resonance is when the inductive reactance and capacitive reactance are at the same value, the point which this occurs depends upon frequency. Therefore the impedance of the circuit and the circuit current are also dependent upon frequency.

Plot a graph of inductive reactance, capacitive reactance and impedance against frequency for the following values.

**Resistance R = 12Ω**

**Inductance L = 0.15H**

**Capacitance C = 100µF**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Frequency | 10Hz | 20Hz | 30Hz | 40Hz | 50Hz | 60Hz | 70Hz |
| Resistance | 12Ω | 12Ω | 12Ω | 12Ω | 12Ω | 12Ω | 12Ω |
| Inductive Reactance |  |  |  |  |  |  |  |
| Capacitive Reactance |  |  |  |  |  |  |  |
| Impedance |  |  |  |  |  |  |  |
| Circuit Current |  |  |  |  |  |  |  |

Take the supply voltage to 100V

Circuit current

(A)

Resistance

Reactance

Impedance

(Ω)

Frequency (Hz)

It should be seem from the graph that there is a point on the graph where the impedance is equal to the resistance; therefore the inductive and capacitive reactances effectively cancel one another out. Also at the same point the circuit current is also at its maximum. Using this maximum value of current find the potential difference (volt drop) across each component.

|  |  |
| --- | --- |
| Voltage across Resistance |  |
| Voltage across Inductance |  |
| Voltage across Capacitance |  |

Then draw the phasor diagram for this particular circuit at resonance.

Conditions of Resonance in a Series Circuit

1. XL=XC Inductive Reactance is equal to Capacitive Reactance

therefore

1. The circuit impedance is at a minimum
2. The circuit current is at a maximum.
3. The voltage across the Inductor is at a maximum
4. The voltage across the Capacitor is at a maximum

It can be seen from the example shown earlier that there is a very large voltage across the inductor and capacitor at resonance. This voltage magnification at resonance is also known as Q factor.

Q-factor = voltage magnification at resonance = voltage across L (or C)

supply voltage V

=  = =

# Problems

Q.18

A circuit having a resistance of 4Ω, an inductance of 0.5H and a variable capacitance connected in series across a 100V, 50Hz supply. Calculate: -

1. The capacitance to give resonance
2. The voltage across the inductance and capacitance
3. The Q factor of the circuit

Q.19

A series circuit consists of a resistance of value 100Ω, an inductor of 10mH, and a capacitor of 100nF is connected to variable frequency supply with an output voltage of 1V. Calculate:-

1. The resonant frequency for the circuit
2. The circuit current at resonance
3. The Q factor of the circuit

# AC Parallel Circuits

# Circuits Containing Resistance and Inductance

In practice most electrical installations consist of a number of circuits connected in parallel to form a network. The branches of the parallel network may consist of one or more components connected in series.

In a parallel circuit the supply voltage is applied to each of the network branches. This is the common factor to each branch of the network and so it is used as the reference when producing phasor diagrams.

IL

IR

V

IR

IL

I

φ

## Problems

Q.20

A circuit of inductance of 20Ω and resistance 30Ω, connected in parallel across a 100V, 50Hz AC supply.

Sketchthe phasor diagram

Calculate the current in each branch

Calculate the circuit current and the phase angle

Q.21

A circuit consists of an inductance of 60Ω and resistance 40Ω, connected in parallel across a 100V, 50Hz supply.

Sketch the phasor diagram

Calculate the current in each branch

Calculate the circuit current and phase angle

# AC Parallel circuits containing Resistance and Capacitance

In a 2 branch parallel circuit containing a resistor R and a capacitor C the current flowing in the resistance IR is in phase with the supply voltage V. The current flowing in the capacitance IC leads the supply voltage by 90°.

IR

IL

I

V

IR

IC



The supply current I is the phasor sum of IR + IC and thus the current I leads the supply voltage by an angle lying between 0° and 90°. This angle will depend upon the values of IR and IC and is shown by the angle α in the phasor diagram.

Note: Leading implies that the current is ahead of the voltage.

IR

IC

I

α

# V IC ILR I VR VL LR- C Parallel a.c. Circuit

In the two-branch circuit containing capacitance C in parallel with inductance L and resistance R, the phasor diagram for the LR branch alone can be shown as:

φ1

VL

VR

ILR

V

The phasor diagram for the C branch alone is shown as:

IC

V

Rotating each and superimposing one upon the other gives the complete phasor diagram shown:

IC

V

ILR

φ1

o

p

q

# Parallel Resonance, Q-factor And Power Factor Improvement

## Parallel Resonance

In the LR-C network resonance occurs when IC = ILRsinφ1

IC

φ1

ILR

Ir = ILRcosφ1

V

ILRsinφ1

VL

VR

I

ILR

IC

V

At this condition the supply current I is in phase with the supply voltage V.

Parallel resonance frequency occurs at

**fr = 1 1 - R2**

**2π √ LC L2**

when R is negligible then

fr = 1

2π√LC

## Current at resonance

At resonance Ir = ILRcosφ1

Where **Ir = VRC**

**L**

The current is minimum at resonance

Since the current at resonance is in phase with the voltage, the impedance of the network acts as a resistance. This impedance is known as the dynamic resistance RD

**RD = L**

**CR**

## Q-factor

Currents higher than the supply current can circulate within the inductor and capacitor branches when the LR-C circuit is at resonance. This occurs because the current leaves the capacitor and establishes a magnetic field in the inductor and vice versa. The Q-factor of a parallel resonant circuit is the ratio of the current circulating in the parallel branches of the circuit to the supply current i.e. the current magnification

Q-factor at resonance = circulating current = IC

supply current I

**Q-factor at resonance = 2πfrL**

**R**

At mains frequencies the Q-factor of a parallel circuit is typically less than 10, but at radio frequencies the Q-factor can be very high.

### Worked Examples

A coil of inductance 0.2 H and resistance 60Ω is connected in parallel with a 20μF capacitor across a 20V, variable frequency supply.

Calculate

1. the resonant frequency
2. the current at resonance
3. the circuit Q-factor at resonance

a) the resonant frequency

fr = 1 1 - R2

2π √ LC L2

fr = 1 1 - 602

2π √ (0.2)(20\*10-6) 0.22

fr = 1 √ 250000 – 90000 = 1 √ 160000

2π 2π

fr = 63.66Hz

b) the current at resonance

Ir = VRC = 20\*60\*(20\*10-6) = 0.12A

L 0.2

c) the circuit Q-factor at resonance

Q-factor at resonance = 2πfrL = 2π\*63.66\*0.2 = 1.33

R 60

# Power in a.c. circuits

Power consumed by a resistor is dissipated in heat and not returned to the source. This is true power. True power is the rate at which energy is used.

Current in an AC circuit rises to peak values and diminishes to zero many times a second. The energy stored in the magnetic field of an inductor, or plates of a capacitor, is returned to the source when current changes direction.

Although reactive components do not consume energy, they do increase the amount of energy that must be generated to do the same amount of work. The rate at which this non-working energy must be generated is called reactive power.

Power in an AC circuit is the vector sum of true power and reactive power. This is called apparent power.

* True power is equal to apparent power in a purely resistive circuit because voltage and current are in phase.
* Voltage and current are also in phase in a circuit containing equal values of inductive reactance and capacitive reactance.
* If voltage and current are 90 degrees out of phase, as would be in a purely capacitive or purely inductive circuit, the average value of true power is equal to zero.

In a purely resistive a.c. circuit the average power dissipated is given by P = VI = I2R = V2/R

In a purely inductive a.c. circuit the average power dissipated is zero

In a purely capacitive a.c. circuit the average power dissipated is zero

To calculate the true power dissipation in an a.c. circuit the following formulae may be used

P = I2R

or P = VI cosφ

or P = VI cosα

This is measured in watts

## Power Factor

Apparent power S = VI

This is measured in volt-amps (VA)

Power Factor = True Power (P)

Apparent Power (S)

Substituting in the equations for true and apparent power we get

Power factor = VI cosφ = cosφ

VI

In a purely resistive circuit, where current and voltage are in phase, there is no angle of displacement between current and voltage. The cosine of a zero degree angle is one. The power factor is one. This means that all energy delivered by the source is consumed by the circuit and dissipated in the form of heat.

In a purely reactive circuit, voltage and current are 90 degrees apart. The cosine of a 90 degree angle is zero. The power factor is zero. This means the circuit returns all energy it receives from the source to the source.

A high power factor reduces the current flowing in a supply system and therefore reduces the cost of cables, switchgear, transformers and generators. Industrial loads such as a.c. motors are essentially inductive and may have a low power factor. One method of improving the power factor is to connect a static capacitor in parallel with the load.

### Worked Examples

Example 1

A circuit has a supply current of 3.757A, a voltage of 240V and circuit phase angle of 37.02° leading. Calculate

* the true power
* apparent power
* power factor

P = VI cosα = 240 \* 3.757 \* cos 37.02° = 720W

S = VI = 240 \* 3.757 = 901.7VA

power factor = P = 720 = 0.8 leading

S 901.7

Example 2

A capacitor C is connected in parallel with a resistor R across a 120V, 200Hz supply. The supply current is 2A at a power factor of 0.6 leading. Determine the values of C and R.

I = 2A

Power factor = cos α = 0.6 leading hence

IC

α = cos-1 0.6 = 53.13° leading

53.13°

IR

from the phasor diagram

IR = I cos 53.13° = 2 \* 0.6 = 1.2A

IC = I sin 53.13° = 2 \* 0.8 = 1.6A

R = V = 120 = 100Ω

IR 1.2

°

IC = V = 2πfCV therefore

XC

C = IC  = 1.6 = 10.61μF

# 2πfV 2π\*200\*120

Example 3

A single-phase motor takes 50A at a power factor of 0.6 lagging from a 240V, 50Hz supply. Determine

1. the current taken by a capacitor connected in parallel with the motor to correct the power factor to unity
2. the value of the supply current after power factor correction

IC

IM = 50A

V=240V, 50Hz

I

C

M

A power factor of 0.6 lagging means that cosφ = 0.6

V= 240V

Hence φ = cos-10.6 = 53.13°

53.13°

IM = 50A

If the power factor is to be improved to unity then the phase difference between the supply current I and voltage V needs to be 0°. For this to be so Ic must equal the length ab, such that the phasor sum of IM and Ic is I.

IC

53.13°

IM

I

V

a

b

ab = IMsin53.13° = 50\*0.8 = 40A

Hence the capacitor current Ic must be 40A for the power factor to be unity.

1. Supply current I = IMcos 53.13°= 50\*0.6 = 30A

Problem

1. A single phase motor takes 30A at a power factor of 0.65 lagging from a 240V, 50Hz supply. Determine

a) the current taken by the capacitor connected in parallel to correct the power factor to unity

b) the value of the supply current after power factor correction

# Filter Networks

A filter is a network designed to pass signals within certain bands with little attenuation, but greatly attenuate the signals within other bands.

There are 4 main types of filters these are:

1. Low pass

- Designed to pass signals at frequencies below a specified cut-off frequency

1. High pass

- Designed to pass signals at frequencies above a specified cut-off frequency

1. Band pass

-Designed to pass signals with frequencies between two specified cut-off frequencies

1. Band stop

-Designed to pass signals with all frequencies except those between two specified cut-off frequencies

For example if we have an a.c. signal is fed into an amplifier the output may include frequencies from DC upwards. If a high pass filter is applied on the output then all the DC will be removed, but higher frequencies will pass through virtually unchanged.

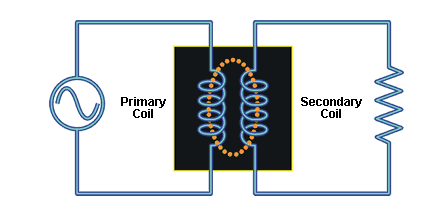
Transformers

A transformer is a device that uses the phenomenon of mutual induction to change the values of alternating currents and voltages. It is a stationary machine i.e. no moving parts.

Transformers vary in size from the miniature units used in electronic applications to the large power transformers used in power stations. The principle of operation is the same for all.

A transformer consists of two electrical circuits linked by a common ferromagnetic core. The primary winding is connected to the supply and the secondary winding is connected to the load.

Principle of Operation



The alternating current applied to the primary winding produces an alternating current through the primary coil. This produces an alternating magnetic flux in the core. In an ideal transformer there are no flux losses and this entire flux links both the primary and secondary coils. Because we have the same flux linking each turn of the primary and secondary coils we must have the same induced e.m.f. produced for each turn., the induced e.m.f. being proportional to the rate of change of flux.

Thus if the primary coil has N1 turns and the secondary coil N2 turns:

Induced e.m.f in primary = N1 \* e.m.f. induced per turn

Induced e.m.f in secondary = N2 \* e.m.f. induced per turn

Hence

Induced e.m.f. in primary = N1

Induced e.m.f. in secondary N2

When the secondary coil is open circuit its terminal voltage V2 is the same as the induced e.m.f. If no load current is taken from the secondary coil then no energy is taken. Therefore, for an ideal transformer, no energy is taken from the primary coil and so there is very little net current in the primary coil. This means that the net e.m.f. in the primary coil is zero, i.e. the induced e.m.f. is equal to but opposing the input voltage.

V1 = N1

V2 N2

When a load is connected across the secondary winding, a current I2 flows. In an ideal transformer losses are neglected and a transformer is considered to be 100% efficient.

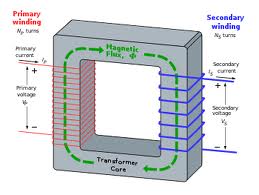
Hence input power = output power

V1I1 = V2I2

And V1 = N1 = I2

V2  N2 I1

The rating of a transformer is stated in terms of the volt-amperes that it can transform without overheating. The transformer rating is either V1I1 or V2I2.

Examples

1. A transformer has 600 primary turns connected to a 1.5kV supply. Determine the number of secondary turns for a 240V output voltage assuming no losses.
2. An ideal transformer with a turns ratio of 2:9 is fed from a 220V supply. Determine its output voltage.

EMF Equation of a transformer

Maximum value of the flux Фm webers

Frequency f hertz

It can be worked out that the emf induced in the primary is:

E1 = 4.44N1f Фm

Similarly the emf induced in the secondary is:

E2= 4.44N2f Фm

Example

1. A single-phase 500V/100V, 50Hz transformer has a maximum core flux density of 1.5T and an effective core cross-sectional area of 50cm2. Determine the number of primary and secondary turns.

Problems

1. A 250kVA, 11000V 400V, 50Hz single-phase transformer has 80 turns on the secondary. Calculate:
2. The approximate values of the primary and secondary currents
3. The approximate number of turns
4. The maximum value of flux
5. A single-phase, 50Hz transformer has 25 primary turns and 300 secondary turns. The cross sectional area of the core is 300cm2. When the primary winding is connected to a 250V supply, determine
6. The maximum value of the flux density in the core
7. The voltage induced in the secondary winding